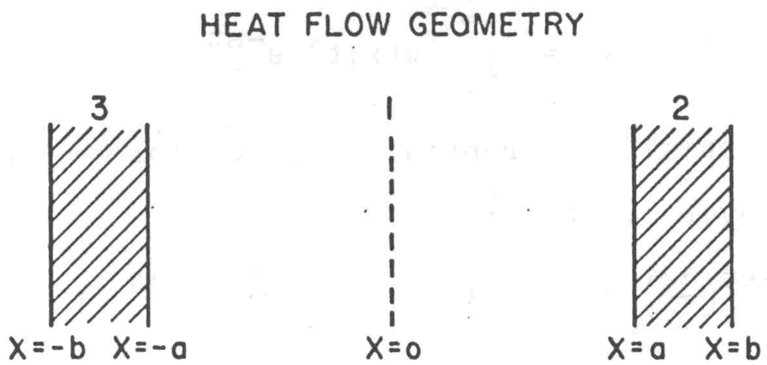

Fig. D.1. Three-slab heat flow geometry.

Boundary conditions

$$\Psi(b,t) = 0, \quad \eta(-b,t) = 0$$

Jump conditions

$$\varphi(a,t) = \Psi(a,t) \qquad \varphi(-a,t) = \eta(-a,t)$$

$$\lambda\varphi_x(a,t) = \Lambda\,\Psi_x(a,t) \qquad \lambda\varphi_x(-a,t) = \Lambda\eta_x(-a,t).$$

(λ, Λ are thermal conductivities.)

Define the Laplace transform of φ by

$$\Phi\,(s,x) = \int_o^\infty \varphi(x,t)\,e^{-st}\,\partial t$$

Multiply the partial differential equation for φ by e^{-st} and integrate over all time:

$$\int_o^\infty e^{-st}\left(\frac{\partial\varphi}{\partial t}\right)_x \partial t = k \int_o^\infty \left(\frac{\partial^2\varphi}{\partial x^2}\right)_t e^{-st}\,dt$$

Integration by parts gives $-\varphi(x,o) + s\Phi = k\,\dfrac{d^2\Phi}{dx^2}$.

We now have an ordinary differential equation for Φ , $\left(\dfrac{d^2}{dx^2} - \dfrac{s}{k}\right)\Phi = -\dfrac{T_1}{k}$. Similar results are obtained for the other regions. The solutions to the differential equations for the Laplace transforms can be expressed:

Region 1, $\Phi(s,x) = A\cosh(\mu x) + B\sinh(\mu x) + \dfrac{T_1}{s}$, $\mu \equiv \left(\dfrac{s}{k}\right)^{1/2}$

Region 2, $\Psi(s,x) = C\cosh(\mu p x) + D\sinh(\mu p x) + \dfrac{T_2}{s}$, $p \equiv \left(\dfrac{k}{\kappa}\right)^{1/2}$

Region 3, $H = E\cosh(\mu p x) + F\sinh(\mu p x) + \dfrac{T_2}{s}$.

After some effort, the coefficients can be found by applying the several conditions of the problem. Then the inversion